## 3.21. MICHELSON'S INTERFEROMETER

It is instrument devised by Michelson in which the amplitude of a light beam from an extended source is divided into two parts of equal intensity by partial reflection and refraction. The parts then reunite to give interference pattern in the field of view (see fig. 3.30)

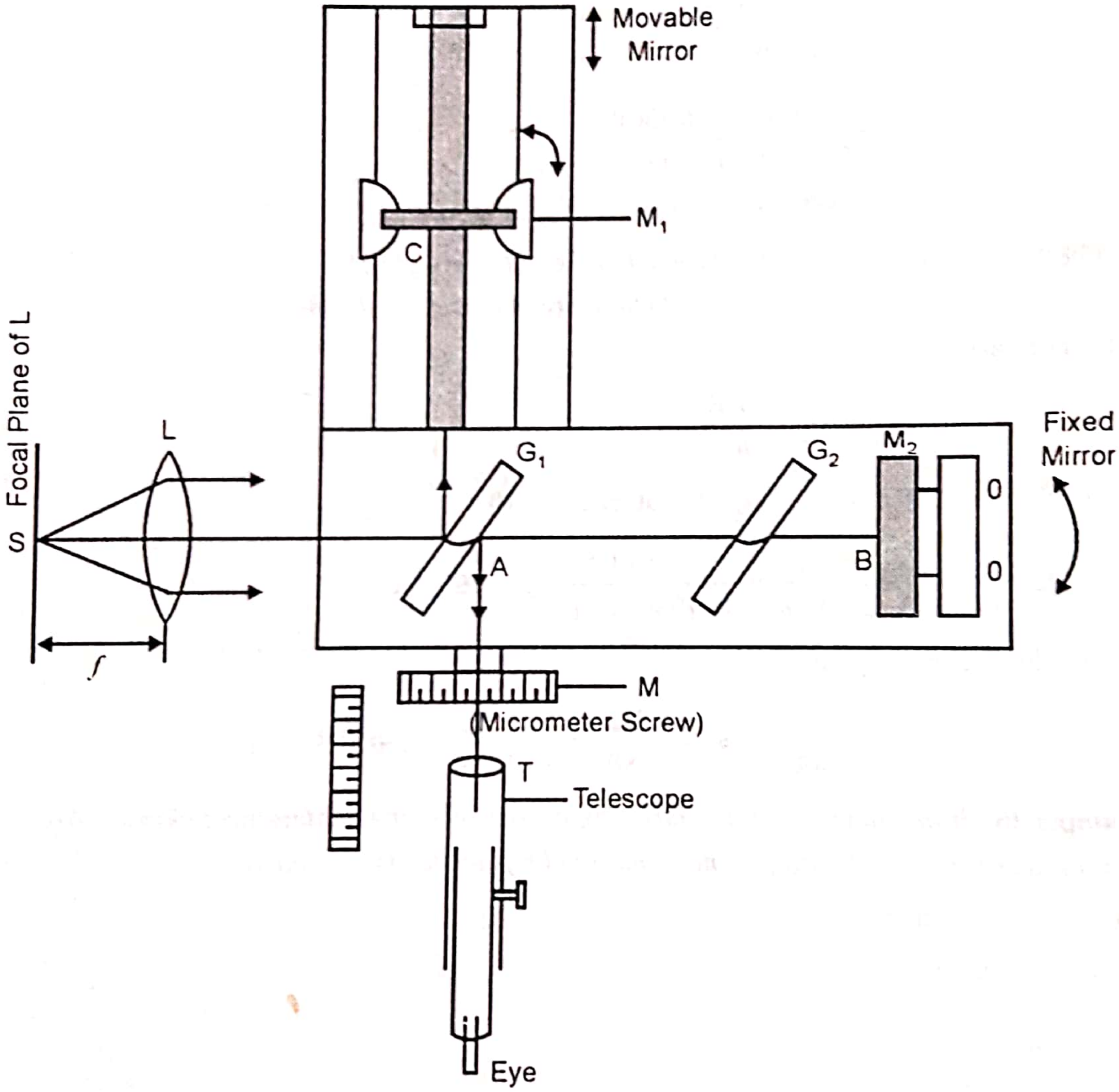


*Fig. 3.30.* Michelson Interferometer

**Construction .** The main **optical parts** consists of two highly polished front silvered plane mirrors $M_1$ and $M_2$ and two plane parallel glass plates $G_1$ and $G_2$ of the same thickness. The plate $G_1$ is half silvered at the back so that incident beam is divided into a reflected and transmitted beam of equal intensity. The plates $G_1$ and $G_2$ are held parallel to each other and are inclined at an angle of 45° to the mirrors $M_1$ and $M_2$. The mirrors $M_1$ and $M_2$ are provided with the levelling screws at the back. With the help of these screws, the mirrors can be made **perfectly perpendicular** to the direction of two beams.† The mirror $M_1$ is mounted on a carriage and can be moved exactly parallel to itself with the help of a micrometer screw M fitted with graduated drum. The count of the screw is $10^{-5}$ cm. (Fig. 3.30). The source of light is a point source S, which is made extended by allowing the light to fall on the collimating lens L. S is to be located on the focal plane of this lens, *i.e.*, SL distance = focal length of L.

The interference bands are observed in the field of view of telescope T.

**Adjustment.** (*i*) The distances of mirrors $M_1$ and $M_2$ are adjusted to be equal from the glass plate $G_1$.

(*ii*) To make the incident light from the source parallel, a thin sheet with a fine hole in it is placed in front of the sources just opposite to the bright part of the flame. The hole, the flame, centre of glass plate and the mirror $M_2$ should be in one level. A convex lens is placed between $G_1$ and the tin sheet. A plane mirror is placed between $G_1$ and the lens normally. The position of the lens is adjusted till the image of the hole falls on the tin sheet . Remove the plane mirror and allow the light to fall on the plate $G_1$. Look through AT. Four images will be visible . The planes of mirrors $M_1$ and $M_2$ are adjusted till the images coincide two by two. If now the tin sheet is removed and two paths are made exactly equal, the field of two will be totally dark. A slight motion of $M_1$ parallel to itself will produce **circular interference fringes.** Bu **tilting** the mirror $M_2$ slightly, the fringes can be made **straight.**

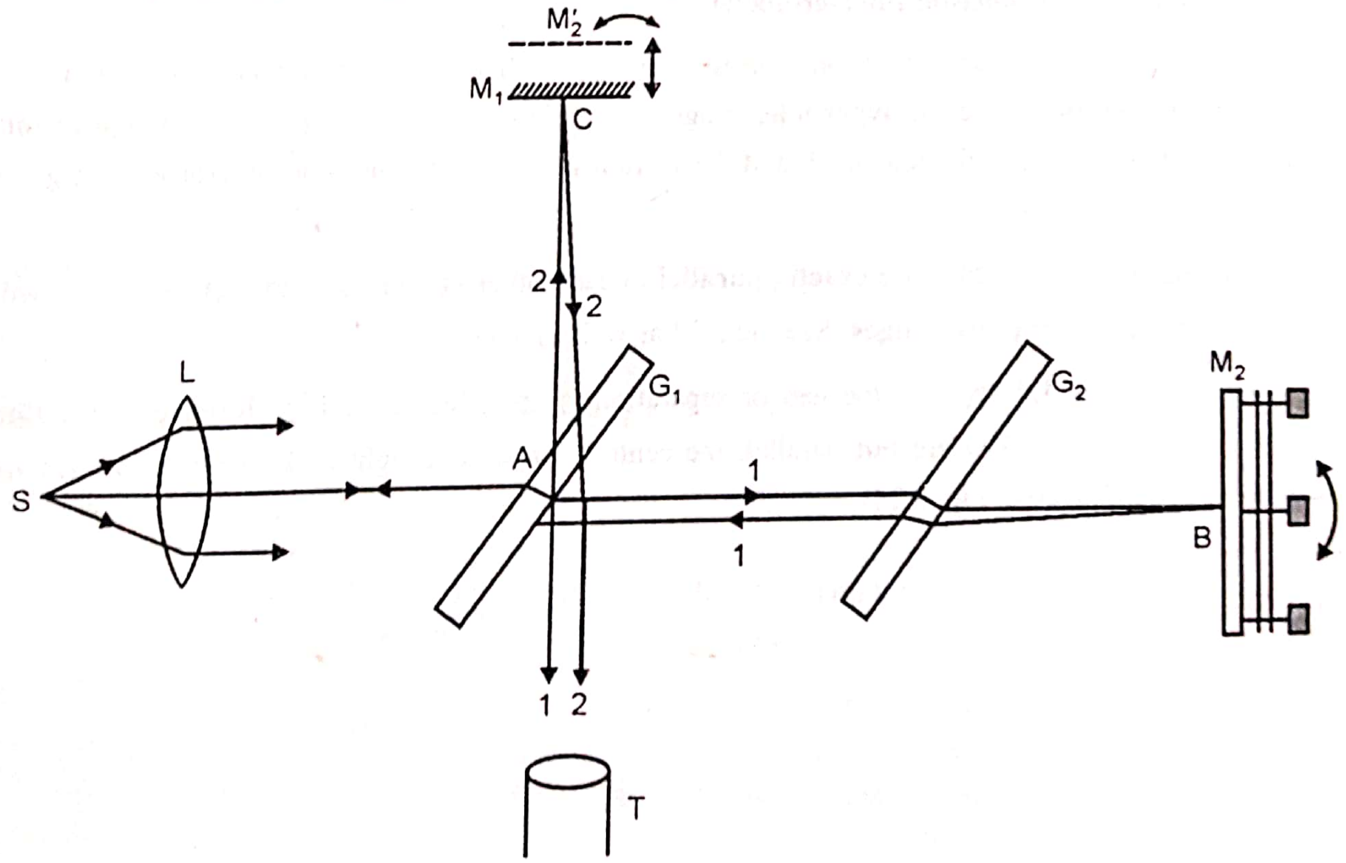


*Fig. 3.31.* The matching of optical paths AB and AC using the compensating plate $G_2$.

† While the mirror $M_2$ is **fixed** and can only be tilted with the help of back screws, the mirror $M_1$ is mobile and can be moved back and forth as well as tilted with the help of back screws

**Working.** Light from the source S is rendered parallel by means of a collimating lens and is made to fall on the glass plate $G_1$. It is partly reflected at the back surface of $G_1$ along AC and partly transmitted along AB. The light AC is received by the plane mirror $M_1$ normally and is reflected back along the same path and comes out along AT. The transmitted beam AB is received by the plane mirror $M_2$ normally and is reflected back and then moves along AT in the Telescope after reflection at the back surface of $G_1$. These two beams, produced from the same source, produce interference under suitable conditions.

It is clear from Fig. 3.30 or Fig. 3.31 that the ray AC traverses the glass plate $G_1$ **thrice** where as ray AB traverses the glass plate **once.** To **compensate** for this, an **exactly similar** glass plate $G_2$ is introduced in the path AB parallel to the glass plate $G_1$. These rays along AB and AC are marked 1 and 2 respectively in Fig. 3.31.

Two waves interfere after travelling distances of the order of few cms; therefore optical path difference may be of many wavelengths so that interference fringes are visible for higher orders.

In case of Newton's rings, Fresnel biprism, the optical path difference is always of wave length's order, so **no** interference fringes are visible for higher orders of path difference.

**Form of fringes with Michelson Interferometer**

Interestingly enough, we can get various forms of fringes with this interferometer. Thus, **straight, circular, parabolic, elliptic,** or even **hyperbolic** fringes are possible depending upon (*i*) the **separation** and (*ii*) the **inclination** between the mirror $M_1$ and the **virtual** image $M_2'$ of the other mirror $M_2$. Of great interest to us are the two cases :

1. When the two, $M_1$ and $M_2'$, are **exactly parallel** to each other *i.e.*, there is no inclination, we will have **concentric, circular fringes** [See Fig 3-32 as well as 3.33]
2. When $M_2'$ coincides $M_1$, *i.e.*, the gap or separation is zero, but there is a **definite inclination** between them , *i.e.*, they are **not parallel**, the central fringe is straight and all other fringes are convex towards it (see. Fig. 3.34)

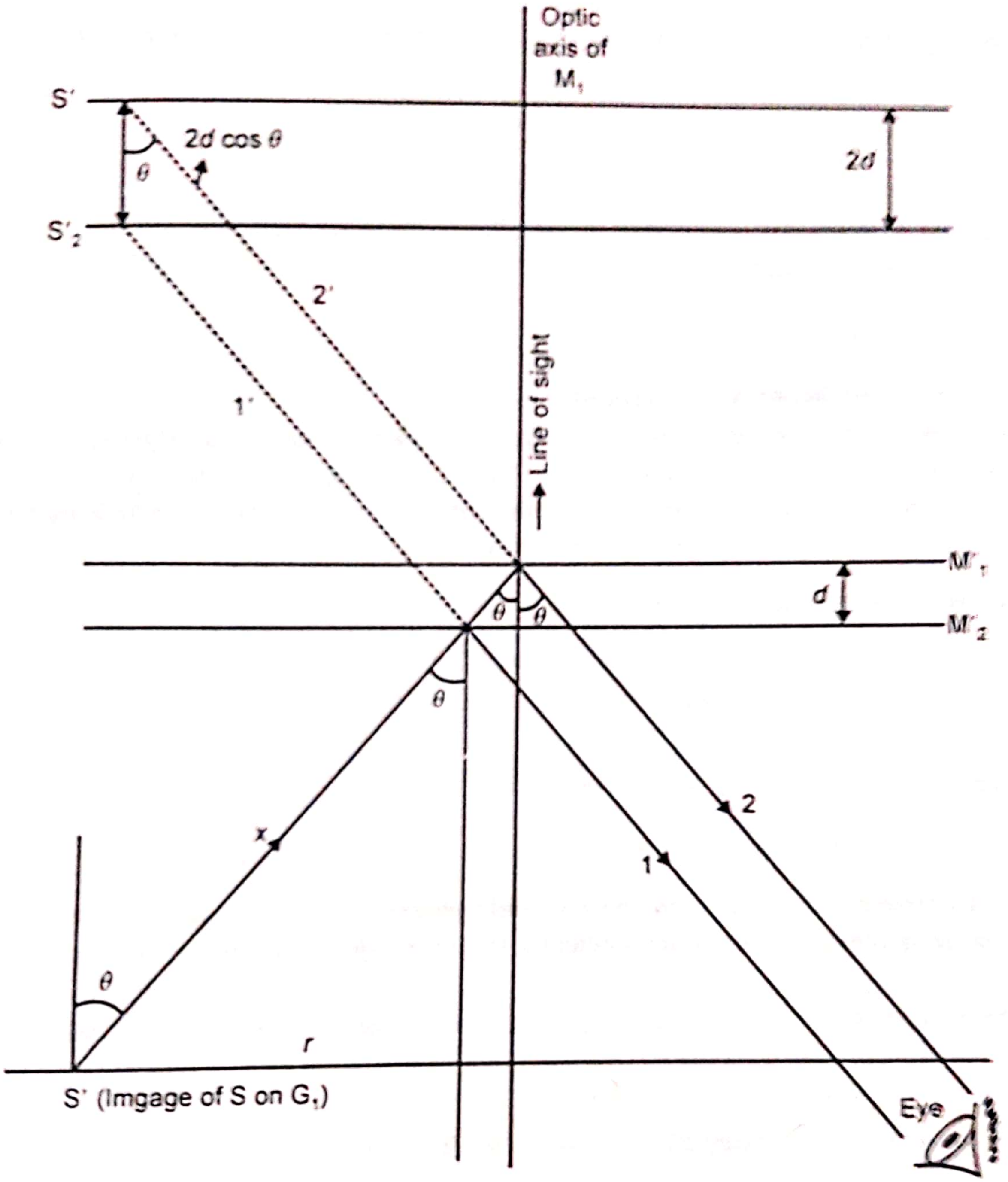


*Fig. 3.32.* Formation of circular fringes, $M_2' \parallel M_1$ and there is a separation $d \neq 0$ between them.

Refering to Fig. 3.32, we note that S' is the image of the given point source S (extended through the use of converging lens L ) formed in the glass plate $G_1$. And $S_1'$ and $S_2'$ are the virtual images of S' in the mirrors $M_1$ and $M_2'$ (*i.e.*, image of $M_2$ in $M_1$). If $M_1$ and $M_2'$ are separated by $d$ (even through their inclination to each other is zero), then $S_1'$ and $S_2'$ are separated by $2d$ . For the rays falling on $M_1$ ($M_2'$) at an angle, the path difference between the rays reaching the eye from $S_1'$ and $S_2'$ is obviously $2d \cos \theta =$ $S_1'S_2' \cos \theta$ (see Fig. 3.32 again). Also, the beam (1) (of fig (3.31)) coming back from $M_2$ and suffering reflection at the plate $G_1$ before going into the telescope involves an **additional** ' phase change of $\pi$ rad or path difference of $\frac{\lambda}{2}$. Thus, the total or **effective** path difference between the two beams (1) and (2)

$$\boxed{\Delta = 2d\cos\theta + \frac{\lambda}{2}} \qquad ...(3.63)$$

And constructive or destructive interference is controlled by the usual condition :

$\Delta = n\lambda$ (constructive interference),

$\Delta = (2n+1)\frac{\lambda}{2}$ (destructive interference)

**Circular fringes in Michelson's Interferometer**

If we look in the direction of the mirror $M_1$, the eye will see the mirror $M_1$ directly and also a **virtual image of** $M_2$ is seen by reflection in the glass plate $G_1$. One of the interfering beams, therefore, comes from $M_1$ and the other appears to come by reflection from the virtual image of mirror $M_2$ which appears to be $M_2'$. These two plates give rise to two virtual sources $S_1$ and $S_2$ as shown in fig. 3.32 below.

So, for bright fringes, $2d\cos\theta + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$, giving

$$\boxed{2d\cos\theta = (2n-1)\frac{\lambda}{2}} \qquad ...(3.64)$$

And for dark fringes, $2d\cos\theta + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$, giving

$$\boxed{2d\cos\theta = n\lambda} \qquad ...(3.65)$$

Thus, for given value of $d$, $n$ and $\lambda$, **the angle $\theta$ will be constant** † so that maximum or minimum lies in the **from of a circle about the foot of perpendicular from the eyes to the mirror,** *i.e.*, the optic axis of the mirror $M_1$

**In the line of sight of eye**, perpendicular to the mirror, the fringes are governed by the relation ($\theta = 0$, $\cos\theta = 1$).

$$2d = n'\lambda \qquad ...(3.66)$$

where $n'$ is an integer. While in any other direction, the relation is

$$2d\cos\theta = n\lambda \qquad (3.67)$$

Fro m eqn (3.65) and (3.66) , we have

$$2d - 2d\cos\theta = (n'-n)\lambda$$

or $$2d(1-\cos\theta) = (n'-n)\lambda$$

or $$(1-\cos\theta) = \frac{m\lambda}{2d} \qquad [n'-n=m]$$

but $$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - ...,$$

$$\therefore \quad 1-\left(1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-\ldots\right)=\frac{m\lambda}{2d}$$

Neglecting $\frac{\theta^4}{4!}$ and higher terms, we have

$$\frac{\theta^2}{2}=\frac{m\lambda}{2d} \text{ or } \theta^2=\frac{m\lambda}{d}$$

or $\boxed{\theta \propto \sqrt{m}}$. ...(3.62)

If $\theta_1, \theta_2, \theta_3$ are the angles for first, second, third,...rings, and $r_1, r_2, r_3$ are their respective radii, then

$$\theta_1=\frac{r_1}{x}, \theta_2=\frac{r_2}{x}, \theta_3=\frac{r_3}{x}, \text{ etc,} \qquad [\because \theta \propto r \propto \sqrt{m} \text{ for } m = \text{integral values}]$$

where $x$ is the distance of $M_2'$ from S′

From above, it is clear that the radii of rings are proportional to the square root of natural numbers. It automatically means that the fringes are **not** equal-spaced.

**▸▸Inbox :** $r \propto \sqrt{m}$ for $m = 1, 2, 3, 4, 5$, so that for simplicity, at once brings out the fact that circular fringes are **not** equidistant as we go away from the central fringe ;

$r_1 \propto 1, r_2 \propto \sqrt{2}=1\cdot414, r_3 \propto \sqrt{3}=1\cdot732, r_4 \propto \sqrt{4}=2$, $r_5 \propto \sqrt{5}=2\cdot236$, and so on .

The spacings gradually **decrease** as higher orders are taken. Also , from eqn (3.65) and (3.66) it follows that $n' > n$ *since* 2d > $2d\cos\theta$. Thus, the order of the fringe at the centre is always higher than that of any other fringe.

**Special Case $d = 0$** When $M_1$ and $M_2'$ are **exactly coincident,** the path difference between rays will be $\frac{\lambda}{2}$, so the field of view will be **dark** Fig. 3.33 (*ii*) . When $M_2'$ is moved either way parallel to itself, widely spaced circular fringes are produced as shown in Fig. 3.33 (*ii*) and (*iii*)

(*i*) (*ii*) (*iii*)

*Fig. 3.33.* Circular concentric fringes and black (dark) field of view for $d = 0$ and the plate $G_1$ unsilvered

Fringes of the type described above, where parallel beams produce interference with a phase difference determined by angle of inclination are known as **Fringes of equal inclination.**

**Localised fringes†**

When the mirrors $M_1$ and $M_2'$are **not exactly parallel,** the fringes are still visible with monochromatic light. In this case, the air film between $M_1$ and $M_2'$ will be **wedge shaped.**

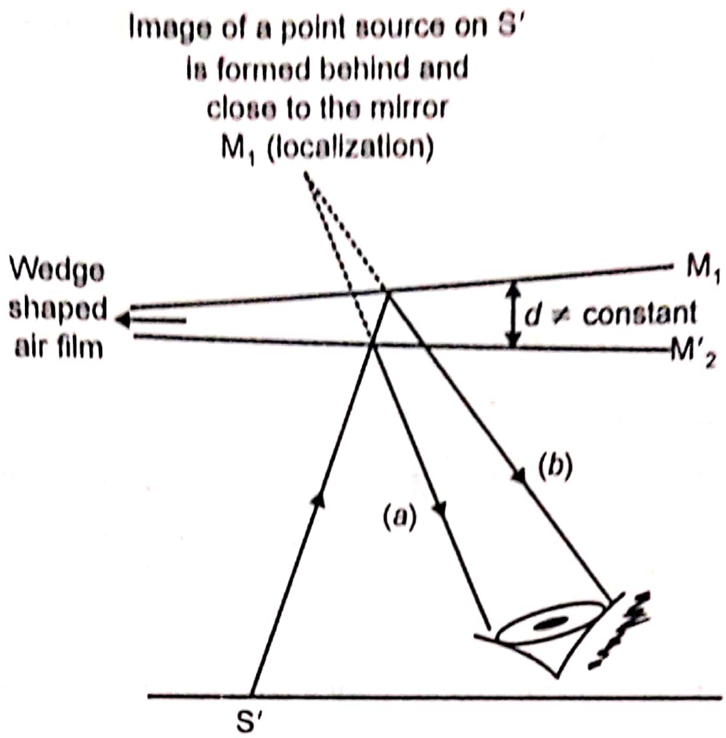


*Fig. 3.34.* Foerays from S' reflected rays (*a*) and (*b*) from $M_2$ and $M_1$ are not parallel but they intersect near $M_1$. Eye sees localized fringes here

With such a film , the locus of the points of equal thickness is a **straight line parallel to the edge of the wedge**. The fringes will be **nearly straight when path difference is very small.** For large value of the distance (path difference) between two mirrors, the fringes are straight as the path difference changes with the angle. *The fringes in general are curved and are convex towards the edge of wedge.* For certain value of separations, fringes observed are shown in Fig. 3.35 (*i*) As $d$ decreases, the fringe system is displaced laterally , a new fringes crossing the centre of field each time changes by $\frac{\lambda}{2}$. The curvature of the fringes goes on decreasing and when path difference becomes zero *i.e.*, when $M_1$ and $M_2$ **cross each other,** the fringes actually become straight Fig 3.35 (*ii*). Beyond this point, they begin to curve in the **opposite** direction as shown in Fig 3.35 (*iii*).

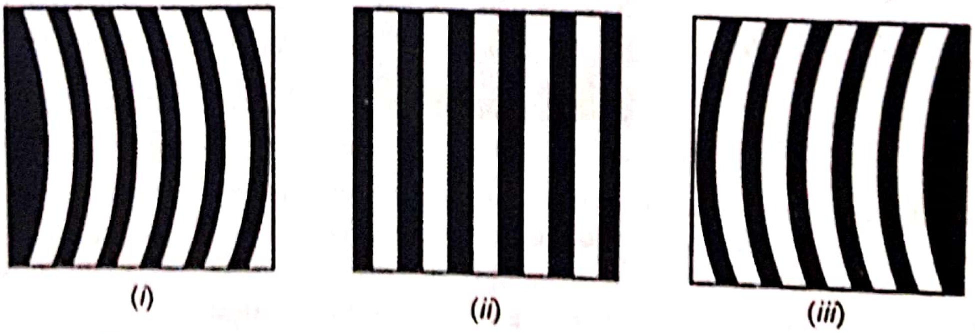


*Fig. 3.35.* Varying curvature of the localized fringes (near $M_1$)

are straight, circular, parabolic or hyperbolic.

## 4.18. APPLICATIONS OF MICHELSON'S INTERFEROMETER

*(i) Determination of wavelength of monochromatic light*

Adjust the Michelson interferometer so that circular fringes are visible in the field of view of telescope. As the mirror $M_2$ is fixed, move mirror $M_1$ so that the entire field of view is dark. In this position the mirrors are equidistant from the beam splitter, *i.e.* $d_1 = d_2$. Thus the distance $d$ will be zero. As the mirror $M_1$ is moved fringes will move across the field of view. For every displacement of mirror $M_1$ by $\frac{\lambda}{2}$, the optical path changes by $\lambda$. Thus each fringe will move to the position previously occupied by the adjacent fringe. This means that one fringe will cross the point of observation (cross wire) in field of view as the mirror moves through a distance $\lambda/2$. If $N$ is the number of fringes which cross the field of view then the distance $d_0$ moved by the mirror is

$$d_0 = \frac{N\lambda}{2}$$

$d_0 = d$

or $$\lambda = \frac{2d_0}{N} = \frac{2d}{N} \qquad ...(4.113)$$

The distance $d_0$ can be measured accurately with the help of micrometer. The number of fringes which cross the centre of field of view can be counted. Thus the wavelength of monochromatic light can be determined using eq. (4.113).

*(ii) Determination of thickness of a thin transparent sheet*

Michelson interferometer is set so that the entire field of view is dark. This means that there is no fringe in the field of view. Now insert a transparent material of thickness $t$ and refractive index $\mu$ in the path of the beam from beam splitter $G_1$ to mirror $M_1$. This will introduce a path difference of $2(\mu - 1)t$ between two interfering beams instead of $(\mu - 1)t$. The reason is that the ray 2 which gives rise to interfering wave 5 will cross the sheet twice (see fig. 3.17). As a result of it the number of fringes will appear in the field of view. The mirror $M_1$ is now moved towards beam splitter till there is no fringe in the field of view. Count the number of fringes which cross the field of view. If $N$ is the number of these fringes, then

$$N\lambda = 2(\mu - 1)t$$

or $$t = \frac{N\lambda}{2(\mu - 1)} \qquad ...(4.114)$$

As $N$ is known $t$ can be calculated.

*(iii) Determination of a small difference in wavelength* / Resolution of spectral lines

Consider a source of light which is a sodium lamp. This source emits light of two wavelengths $\lambda_1$ (5890 Å) and $\lambda_2$ (5896 Å) which differ slightly. The difference in these wavelengths $|\lambda_1 - \lambda_2|$ can be determined using Michelson interferometer.

Michelson interferometer is adjusted to obtain the fringes. The two wavelengths $\lambda_1$ and $\lambda_2$ will produce their separate fringe patterns. When the interference pattern is seen through a telescope focussed for infinity, it will be a superposition of two fringe patterns. At positions where the bright fringe of one pattern falls on the bright fringe of other pattern the resultant intensity will be high. Thus the fringe visibility will be maximum. The fringe visibility will be minimum when a bright fringe of one system falls on the dark fringe of other system. Adjust the position of the mirror $M_1$ so that a fringe of maximum visibility is formed at the centre of field of view. Suppose the bright fringe of wavelength $\lambda_1$ falls on the bright fringe of wavelength $\lambda_2$. If $d$ is the distance of mirror $M_1$ from beam splitter then

$$2d = \left(n_1 + \frac{1}{2}\right)\lambda_1 \qquad \text{...(4.115)}$$

and

$$2d = \left(n_2 + \frac{1}{2}\right)\lambda_2 \qquad \text{...(4.116)}$$

These equations may be written as

$$n_1 = \frac{2d}{\lambda_1} - \frac{1}{2}$$

and

$$n_2 = \frac{2d}{\lambda_2} - \frac{1}{2}$$

Subtracting these equations we obtain

$$n_1 - n_2 = 2d\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \qquad \text{...(4.117)}$$

The mirror $M_1$ is now moved such that a dark fringe of $\lambda_1$ falls on the bright fringe of $\lambda_2$ and conversely. The superposition fringe system will now disappear. This situation will occur when $n_1$ changes to $n_1 + \frac{1}{2}$. The condition corresponding to eq. (4.117) will have the form

$$n_1 + \frac{1}{2} - n_2 = 2d'\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \qquad \text{...(4.118)}$$

where $d'$ is the distance of mirror $M_1$ from beam splitter. Subtract eq. (4.117) from eq. (4.118) to obtain

$$\frac{1}{2} = 2(d' - d)\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\frac{1}{4} = (d' - d)\left(\frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}\right) \qquad \text{...(4.119)}$$

where $d' - d$ is the distance moved by the mirror $M_1$ from position of maximum visibility to disappearance of fringe system. If we denote $d' - d$ by $x$ we may write

$$\lambda_2 - \lambda_1 = \frac{\lambda_1 \lambda_2}{4x} \qquad \text{...(4.120)}$$

If $d_0$ is the displacement of movable mirror between two consecutive positions of maximum intensity or two consecutive disappearance then

$$d_0 = 2x$$

or

$$x = \frac{d_0}{2}$$

Substitute this value in eq. (4.120) to obtain

$$\lambda_2 - \lambda_1 = \frac{\lambda_1 \lambda_2}{2d_0} \quad ...(4.121)$$

If we take $\lambda$ as the mean wavelength of two wavelengths $\lambda_1$ and $\lambda_2$, we may write

$$\lambda_2 - \lambda_1 = \frac{\lambda^2}{2d_0} \quad ...(4.122)$$

Thus the difference in wavelength $\lambda_2 - \lambda_1$ can be calculated.

## 4.19. NON-LOCALIZED AND LOCALIZED FRINGES

To observe the fringes formed it is necessary to know the region where the fringes are produced. Depending on the location of fringes, they are divided into two types.

(*a*) *Non-localized fringes.* Fringes which are not confined to some small region of space are known as non-localized fringes. For example in Young's double slit experiment the fringe pattern can be obtained any where in front of the sources (slits). Thus these fringes are non-localized. This sort of fringes are generally produced by small sources such as point or line sources.

(*b*) *Localized fringes.* Fringes which are observable only over a particular surface are called localized fringes. Fringes produced in thin film are localized either near a film or at infinity. This type of fringe system is produced by broad source as well as by a point source.

## 3.10. FRESNEL'S BIPRISM

Fresnel used a biprism to obtain coherent sources by refraction to produce interference pattern. Fresnel's biprism ABC consists of **two acute** angled prisms with their bases in contact, forming **a single obtuse-angled** prism (obtuse angle of about 179°). In actual practice, the prism is ground from a single optically true glass plate. The acute angles at B and C are about 30 minutes each (Fig. 3.7).

The biprism produces two virtual images $S_1$ and $S_2$ of a fine slit S illuminated by a monochromatic source of light by refraction at the upper and lower halves of the prism as shown in Fig. 3.7.

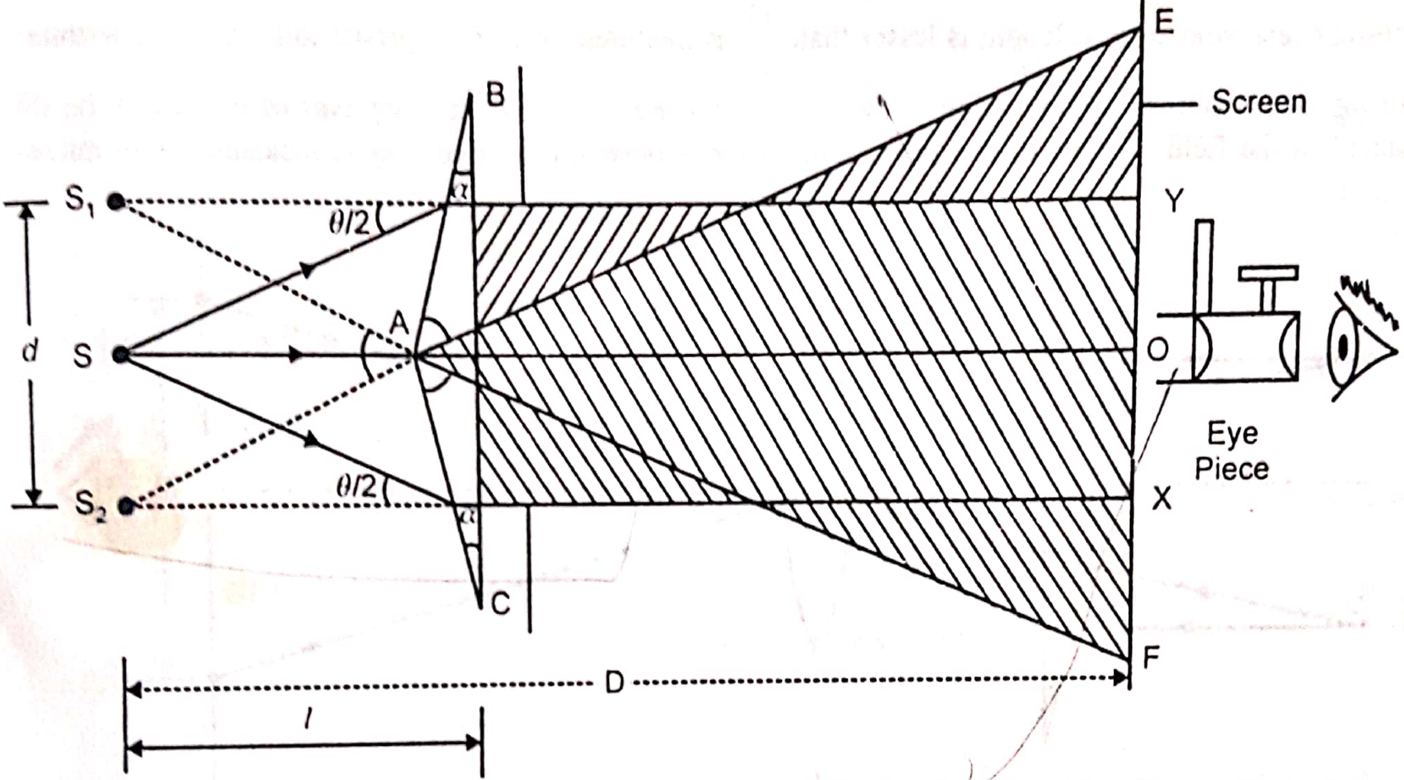


*Fig. 3.7.* A Fresnel's bi-prism in action ; fringes are observed in the region XY. $S_1$ and $S_2$ are two virtual images of S and serve as two coherent sources.

Since, these virtual sources $S_1$ and $S_2$ are derived from the same parent sources S, they act as coherent sources.

The source $S_1$ illuminates the part YF of the screen while $S_2$ illuminates the part EX. Alternate dark and bright interference fringes are observed in the common area XY of the screen. Monochrome sources give bright and dark fringes, while polychromatic ones give a few coloured fringes.

**Experimental arrangement.**

*Adjustment.* A narrow adjustable slit S, illuminated by sodium light, a biprism, and a micro-meter eye-piece are arranged in uprights on the bed of the optical bench in the same straight line and at the same height. The slit is made vertical and parallel to the edge of biprism by rotating it in its own plane.

The biprism is moved parallel to the axis of the optical bench till on looking through it, two equally bright vertical sources $S_1$ and $S_2$ are seen. The eye-piece is moved at right angles to the length of the bench till overlapping region is brought into field of view. The slit is made narrow till the fringes appear in the focal plane of the eye-piece.

(*i*) *Determination of fridge width* ($\beta$). To determine the fringe width, the fringes are first obtained in the field of view of the micro-meter eye-piece. The vertical wire is made to coincide with the centre of bright fringe. The position of cross-wire is read on the scale, considering it as zero position. The micro-meter screw is moved side ways and number of fringes (bright) moved are counted. The position of the eye-piece is again read on the scale. Then

$$\beta = \frac{\text{Total distance moved}}{\text{No. of fringes moved across the cross wire}}.$$

(*ii*) *Determination of D.* The distance (D) between the slit and the focal plane of the eye-piece is found out by noting their position on the scale of the bench. The index correction is to be applied to get correct D.

(*iii*) *Distance between two sources (d).* To determine *d*, the distance between two virtual sources $S_1$ and $S_2$, a convex lens whose focal length is lesser than $\frac{D}{2}$ is mounted between biprism and eye-piece **without disturbing** their positions. The position of the lens is adjusted as at $L_1$ till a sharp pair of images of the slit is obtained in the field of view of eye-piece. The distance between the two slits is measured with micro-meter screw.

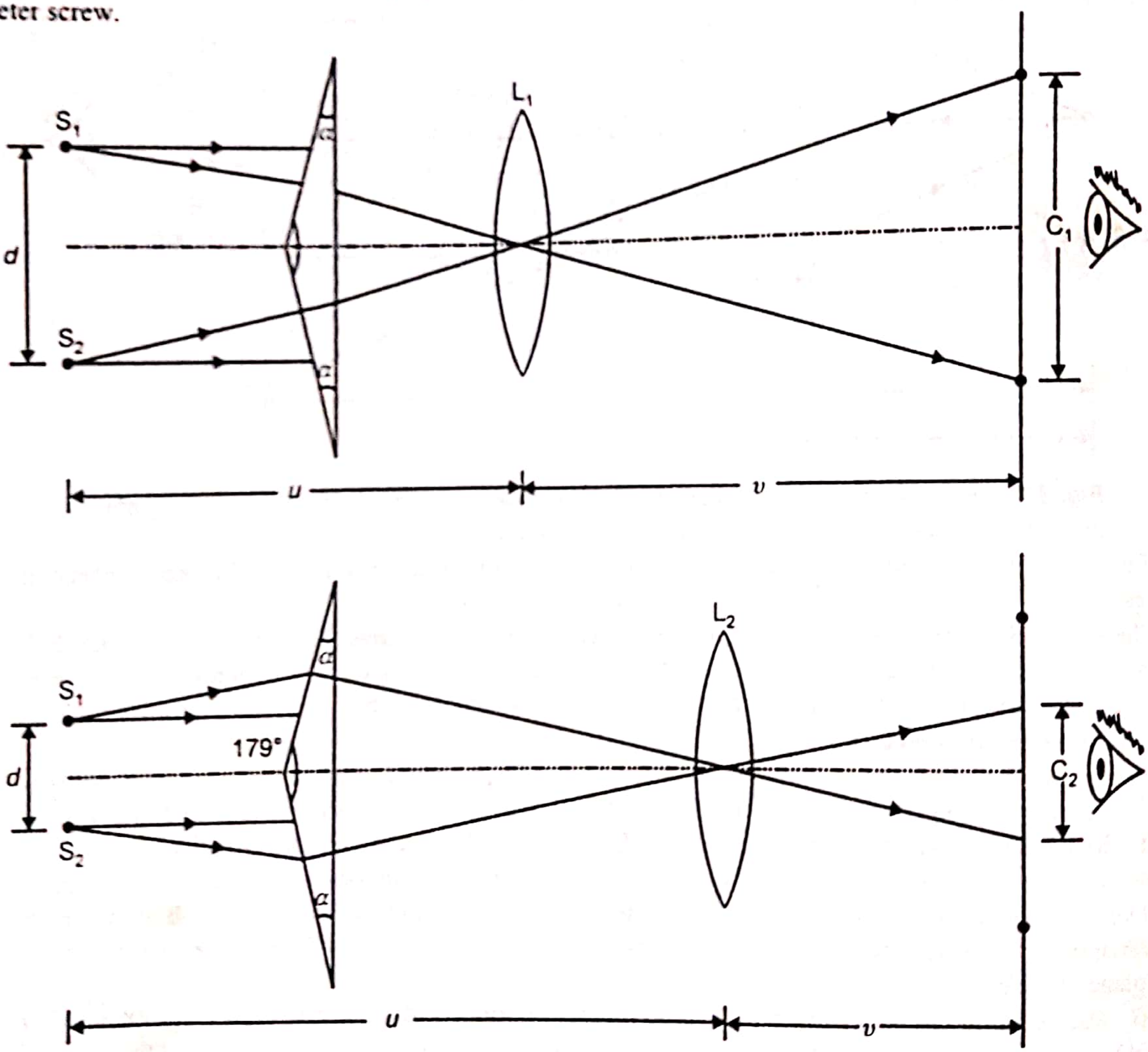


***Fig. 3.8.*** Finding "*d*" by displacement method, using a lens L in two positions $L_1$, $L_2$ giving identical images of the sources $S_1$ and $S_2$ without disturbing anything else.

Let it be $c_1$.

The lens is moved to position $L_2$ till again a pair of sharp images of slit is obtained. Let the distance between them be $C_2$.

In the first case, $$\frac{c_1}{d} = \frac{v}{u}. \qquad ...(3.14)$$

In the second case, $$\frac{c_2}{d} = \frac{u}{v}. \qquad ...(3.15)$$

Thus ; $$\frac{c_1}{d} \cdot \frac{c_2}{d} = \frac{v}{u} \times \frac{u}{v} = 1, \text{ so that}$$

$$d^2 = c_1 c_2 \text{ or } \boxed{d\sqrt{c_1 c_2}.} \qquad ...(3.16)$$

This displacement method was given by Glaze brook.

As $\beta$, D, and $d$ are all known, the wavelength $\lambda$ can be calculated form eqn. (3.11), the relation $\lambda = \dfrac{B \cdot d}{D}$

... We know that the deviation ($\delta$) produced by a small angled prism is given by